on the Basis of an Energy Equation," *Teoreticheskie Osnovy Khimicheskoi Tekhnologii*, 5, No. 4, 564 (1971).
Erdelyi, A., *Higher Transcendental Functions*, Vol. 2, p. 135, Vol. 1, p. 101, McGraw-Hill, New York (1953).
Genkin, V. S., V. V. Dil'man, and S. P. Sergeev, "The Distribution of a Gas Stream Over the Height of a Catalyst Bed in a Radial Contact Apparatus," *Intern. Chem. Eng.*, 13, 24 (1973).
Kaye, L. A., "Fluid Distribution in Radial Flow Vapor Phase, Fixed Bed Reactors," paper presented at 71st AIChE Annual Meeting, Miami Beach, Fla. (Nov., 1978).
Lamba, H. S., and M. P. Dudukovic, "Analysis of the Radial Flow Fixed Bed Reactor," paper presented at 80th AIChE National Meeting, Boston, Mass. (Sept., 1975).
Levenspiel, O., *Chemical Reaction Engineering*, p. 177, Wiley, New York (1972).

Manuscript received March 13, 1978; revision received September 1, and accepted September 6, 1978.

Safety and Reliability Synthesis of Systems with Control Loops

A method of a reliability synthesis for a system with control loops is proposed by introducing a concept which we call a critical transition set. The set is an extended set of an exact failure mode and is important in that each occurrence of the system failure has to correspond to a mode in the set. This means that the system can be improved in such a way as to eliminate all the dominant modes in the set.

First, the system diagram is obtained by connecting outputs of components to the inputs of succeeding components. Time delays are introduced in the feedback loops to represent the internal system state (memory). Then, the components are modeled by decision tables. The critical transition set can be obtained easily by simple tabular manipulations once the system failure is defined. Finally, the system reliability and availability improvements are made based on the set.

HIROMITSU KUMAMOTO
and
ERNEST J. HENLEY

Department of Chemical Engineering
University of Houston
Houston, Texas 77004

SCOPE

This paper develops a new method for obtaining reliability and safety parameters for process systems containing control loops. It is based on the construction of decision tables for the individual components in a process flow sheet, followed by a series of tabular manipulations to eliminate the internal variables and yield the critical transition modes. These are analogous to cut sets and can be used to obtain reliability and safety parameters. Systems can be upgraded and improved by eliminating one-event critical modes and/or modes in which the combined probabilities of failure are undesirably high.

Feedback and feed forward control loops are handled by introducing time delays which represent internal memory. Given an adequate library of component decision tables, the synthesis of large systems can be readily automated, the safety analyst being relieved to the tedium of constructing system fault trees.

CONCLUSIONS AND SIGNIFICANCE

The construction of fault trees to obtain top-event parameters such as reliability, availability, and expected number of failures is a well-established practice in the nuclear and aerospace industries which is being widely adopted in the process industries. Their construction is time consuming; several man years are required to produce a tree for a nuclear power plant. Furthermore, the trees become extremely complicated, errors frequently occur, and the work is tedious and often nonproductive. Another problem is that systems containing control loops are not analyzable by presently available techniques because they lead to logic complexities, problems of state history, and, ultimately, to trees that are not coherent and for which top-event probabilities can not be rigorously calculated.

What is required is a computer based synthesis procedure by which the top-event probabilities for a complicated process flow sheet can be synthesized from "Mini-Fault Trees" which represent the components (including control loops). One such method, based on digraphs, has recently been proposed. This, however, is heuristic in nature and leads to fault trees which are difficult to interpret quantitatively or qualitatively and are not necessarily correct.

In this paper we propose a method for synthesizing system failure modes which is based on decision tables for the individual components. It produces, instead of fault trees, a table of critical transition modes which represent the unique system failure modes. These modes can be used, like cut sets, for improving system safety and for calculating top-event probabilities. The methodology lends itself readily to computer automation.

Correspondence concerning this paper should be addressed to Ernest J. Henley. Hiromitsu Kumamoto is on leave from Kyoto University, Kyoto 606, Japan.

Control loops make the construction of fault trees extremely difficult because of the system logical complexities (Fussell, 1973, 1976); Kumamoto, 1977; Lapp, 1977). For example:

1. The order of component failures becomes important. A system fails dangerously because a failure in a temperature controller causes overheating. Suppose the system has a temperature sensitive cutoff switch on its heater. If the controller fails before the cutoff switch, no accident results. If the switch fails and then the controller fails, the result may be serious.

2. The occurrence of the system failure is dependent on the internal system state, since outputs of feedback controllers are functions of the system state. In a cooling system, a cooling water control valve reversed action is serious when the system state, that is, the temperature at the exit of the heat exchanger, is high. The reversed action causes no failure when the temperature is low because the cooling water increases by the action. It should be noted that the system state is a complicated function of histories of disturbances and component failures and the initial system state. This makes the situation more difficult.

Further, even if the construction of the fault tree is possible, the tree becomes at best very complicated and, at worst, noncoherent or nonassociate. Therefore, it is doubtful that fault trees are useful in system reliability analysis or synthesis where there are complicated logic structures. New approaches have to be developed.

In this paper, we propose a method of a reliability synthesis for a system with the control loops. We introduce a concept which we call a critical transition set. It is a set of (component) failure modes and is important in that each occurrence of the system failure has to correspond to a mode in the set. This means that we can improve the system in such a way as to eliminate all the dominant modes in the critical transition set. The resulting system is guaranteed to have no dominant modes.

First, the system is represented by connecting outputs of components to the input of succeeding components. Time delays are introduced in the feedback loops to represent the internal system state (memory). Then, the components are modeled by decision tables describing each component's outputs in terms of all the possible inputs to the component (Pollack, 1971; Salem, 1977). The critical transition set can be obtained easily by simple tabular manipulations once the system failure is defined. Finally, the system reliability and availability improvements are made based on the set.

OBTAINING THE CRITICAL TRANSITION SET

System Description

Consider a cooling water system shown in Figure 1. The system description of Figure 1 can be obtained easily by connecting inputs and outputs of components in the system. Component internal modes such as normal, reversed, or broken are considered as inputs to the component.

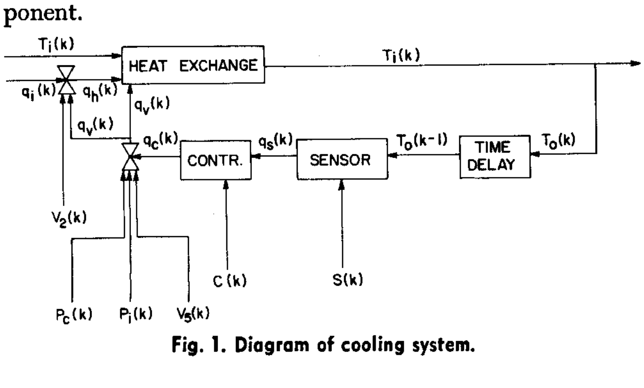

Fig. 1. Diagram of cooling system.

TABLE 1. TOP EVENT TABLE

$T_o(k)$
2

TABLE 2. DECISION TABLE OF CUTOFF VALVE

$V_2(k)$	$q_V(k)$	$q_i(k)$	$q_h(k)$
N	−2	−2	−2
N	−2	0	−2
N	−2	1	−2
N	−1	−2	−2
N	−1	0	0
N	−1	1	1
N	0	−2	−2
N	0	0	0
N	0	1	1
N	1	−2	−2
N	1	0	0
N	1	1	1
R	−2	−2	−2
R	−2	0	0
R	−2	1	1
R	−1	−2	−2
R	−1	0	0
R	−1	1	1
R	0	−2	−2
R	0	0	0
R	0	1	1
R	1	−2	−2
R	1	0	0
R	1	1	1

TABLE 3. SIMPLIFIED DECISION TABLE OF CUTOFF VALVE

$V_2(k)$	$q_V(k)$	$q_i(k)$	$q_h(k)$
—	—	−2	−2
N	−2	—	−2
R	−2	0	0
R	−2	1	1
—	−1	0	0
—	−1	1	1
—	0	0	0
—	0	1	1
—	1	0	0
—	1	1	1

Symbols

The following symbols are used to characterize the condition of hardware, flows, and faults: —: don't care, 0: normal, 1: high, 2: too high, −1: low, −2: too low, *N*: normal, *R*: reversed installation, and *B*: broken (stuck).

Each system variable in the flow sheet is assumed to take the following sets of values: $q_i \epsilon \{-2, 0, 1\}$, $V_2 \epsilon \{N, R\}$, $q_V \epsilon \{-2, -1, 0, 1, 2\}$, $q_h \epsilon \{-2, 0, 1\}$, $T_i \epsilon \{0, 1, 2\}$, $T_o \epsilon \{-1, 0, 1, 2\}$, $S \epsilon \{N, B\}$, $q_S \epsilon \{-1, 0, 1, 2\}$, $C \epsilon \{N, R, B\}$, $q_C \epsilon \{0, 1\}$, $V_5 \epsilon \{N, R\}$, $p_i \epsilon \{0, -2\}$, and $p_C \epsilon \{0, -2\}$.

Classification of Variables

A primary variable is a variable which is an input to a component from outside of the system. On the other hand, an output of a component other than time delay is called an intermediate variable. The output of the delay is a state (memory) variable of the system.

For the system in Figure 1, we have the following classification:

Table 4. Simplified Decision Table of Heat Exchanger

$T_i(k)$	$q_V(k)$	$q_h(k)$	$T_o(k)$
0	−2	−2	0
0	−2	0	2
0	−2	1	2
0	−1	−2	−1
0	−1	0	1
0	−1	1	2
0	0	−2	−1
0	0	0	0
0	0	1	1
0	1	−2	−1
0	1	0	−1
0	1	1	0
1	−2	−2	0
1	−2	0	2
1	−2	1	2
1	−1	−2	0
1	−1	0	2
1	−1	1	2
1	0	−2	−1
1	0	0	1
1	0	1	2
1	1	−2	−1
1	1	0	0
1	1	1	1
2	−2	−2	1
2	—	0	2
2	—	1	2
2	−1	−2	0
2	0	−2	0
2	1	−2	−1

Table 5. Simplified Decision Table of Sensor

$S(k)$	$T_o(k-1)$	$q_S(k)$
N	−1	−1
—	0	0
N	1	1
N	2	2
B	—	0

Table 6. Simplified Decision Table of Controller

$C(k)$	$q_S(k)$	$q_C(k)$
N	−1	0
—	0	0
N	1	1
N	2	1
R	−1	1
R	1	0
R	2	0
B	—	0

Table 7. Simplified Decision Table of Cooling Water Valve (Air to Open)

$p_C(k)$	$V_5(k)$	$q_C(k)$	$p_i(k)$	$q_V(k)$
0	—	0	0	0
0	N	1	0	1
0	R	1	0	−1
0	N	—	−2	−2
0	R	—	−2	1
−2	—	—	—	−2

Table 8. Rewriting the Top-Event Table

$T_i(k)$	$q_V(k)$	$q_h(k)$	$T_o(k)$
0	−2	0	2
0	−2	1	2
0	−1	1	2
1	−2	0	2
1	−2	1	2
1	−1	0	2
1	−1	1	2
1	0	1	2
2	—	0	2
2	—	1	2

Table 9. Rewriting the Top-Event Table

		$q_h(k)$			
$T_i(k)$	$q_V(k)$	$V_2(k)$	$q_V(k)$	$q_i(k)$	$T_o(k)$
0	−2	R	−2	0	2
0	−2	R	−2	1	2
0	−1	—	−1	1	2
1	−2	R	−2	0	2
1	−2	R	−2	1	2
1	−1	—	−1	0	2
1	−1	—	−1	1	2
1	0	—	0	1	2
2	—	R	−2	0	2
2	—	—	−1	0	2
2	—	—	0	0	2
2	—	—	1	0	2
2	—	R	−2	1	2
2	—	—	−1	1	2
2	—	—	0	1	2
2	—	—	1	1	2

Table 10. Rewriting the Top-Event Table

$T_i(k)$	$q_V(k)$	$V_2(k)$	$q_i(k)$	$T_o(k)$
0	−2	R	0	2
0	−2	R	1	2
0	−1	—	1	2
1	−2	R	0	2
1	−2	R	1	2
1	−1	—	0	2
1	−1	—	1	2
1	0	—	1	2
2	−2	R	0	2
2	−1	—	0	2
2	0	—	0	2
2	1	—	0	2
2	−2	R	1	2
2	−1	—	1	2
2	0	—	1	2
2	1	—	1	2

Primary variable: $q_i(k)$, $V_2(k)$, $T_i(k)$, $S(k)$, $C(k)$, $V_5(k)$, $p_i(k)$, $p_C(k)$.
Intermediate variable: $q_V(k)$, $q_h(k)$, $q_C(k)$, $T_o(k)$.
State variable: $T_o(k-1)$.

Definition of System Failure

A system failure is defined by the event that an intermediate variable takes a prespecified value. The system failure of the cooling system in Figure 1 is the event that the temperature $T_o(k)$ is too high. The failure is summarized by Table 1, a top-event table.

TABLE 11. CRITICAL TRANSITION TABLE

Mode No.	$T_i(k)$	$p_c(k)$	$V_5(k)$	$C(k)$	$S(k)$	$p_i(k)$	$V_2(k)$	$q_i(k)$	$T_o(k-1)$	$T_o(k)$	N
1	0	0	N	—	—	−2	R	0	—	2	2
2	0	−2	—	—	—	—	R	0	—	2	2
3	0	0	N	—	—	−2	R	1	—	2	3
4	0	−2	—	—	—	—	R	1	—	2	3
5	0	0	R	N	N	0	—	1	1	2	2
6	0	0	R	N	N	0	—	1	2	2	2
7	0	0	R	R	N	0	—	1	−1	2	3
8	1	0	N	—	—	−2	R	0	—	2	3
9	1	−2	—	—	—	—	R	0	—	2	3
10	1	0	N	—	—	−2	R	1	—	2	4
11	1	−2	—	—	—	—	R	1	—	2	4
12	1	0	R	N	N	0	—	0	1	2	2
13	1	0	R	N	N	0	—	0	2	2	2
14	1	0	R	R	N	0	—	0	−1	2	3
15	1	0	R	N	N	0	—	1	1	2	3
16	1	0	R	N	N	0	—	1	2	2	3
17	1	0	R	R	N	0	—	1	−1	2	4
18	1	0	—	N	N	0	—	1	−1	2	2
19	1	0	—	—	—	0	—	1	0	2	2
20	1	0	—	B	—	0	—	1	—	2	3
21	1	0	—	—	B	0	—	1	—	2	3
22	1	0	—	R	N	0	—	1	1	2	3
23	1	0	—	R	N	0	—	1	2	2	3
24	2	0	R	N	N	0	—	0	1	2	2
25	2	0	R	N	N	0	—	0	2	2	2
26	2	0	R	R	N	0	—	0	−1	2	3
27	2	0	—	N	N	0	—	0	−1	2	1
28	2	0	—	—	—	0	—	0	0	2	1
29	2	0	—	B	—	0	—	0	—	2	2
30	2	0	—	—	B	0	—	0	—	2	2
31	2	0	—	R	N	0	—	0	1	2	2
32	2	0	—	R	N	0	—	0	2	2	2
33	2	0	N	N	N	0	—	0	1	2	1
34	2	0	N	N	N	0	—	0	2	2	1
35	2	0	N	R	N	0	—	0	−1	2	2
36	2	0	R	—	—	−2	—	0	—	2	3
37	2	0	N	—	—	−2	R	0	—	2	3
38	2	−2	—	—	—	—	R	0	—	2	3
39	2	0	N	—	—	−2	R	1	—	2	4
40	2	−2	—	—	—	—	R	1	—	2	4
41	2	0	R	N	N	0	—	1	1	2	3
42	2	0	R	N	N	0	—	1	2	2	3
43	2	0	R	R	N	0	—	1	−1	2	4
44	2	0	—	N	N	0	—	1	−1	2	2
45	2	0	—	—	—	0	—	1	0	2	2
46	2	0	—	B	—	0	—	1	—	2	3
47	2	0	—	—	B	0	—	1	—	2	3
48	2	0	—	R	N	0	—	1	1	2	3
49	2	0	—	R	N	0	—	1	2	2	3
50	2	0	N	N	N	0	—	1	1	2	2
51	2	0	N	N	N	0	—	1	2	2	2
52	2	0	N	R	N	0	—	1	−1	2	3
53	2	0	R	—	—	−2	—	1	—	2	4

Component Description by Decision Tables

A component is modeled by a decision table describing the component's outputs in terms of all the possible input combinations to the component. For example, the cutoff valve in Figure 1 is assumed to be modeled by Table 2. The input variables are $V_2(k)$, $q_V(k)$, and $q_i(k)$, and the output variable is $q_h(k)$. Note that the reversed action, that is, a complete valve opening, occurs only in the case of $q_V(k) = -2$ when the cutoff valve operates. Table 2 can be simplified, resulting in Table 3.

In the same way as for the cutoff valve, we obtain the component descriptions given by Tables 4 through 7.

Obtaining Critical Transition Set

We can eliminate the intermediate variables from the top-event table. This is illustrated by example. The generalization is straightforward.

The top-event Table 1 has the intermediate variable $T_o(k)$. Identify the table having $T_o(k)$ as an output column. Table 4 is identified. Search rows having $T_o(k) = 2$ in this table. Replace, in Table 1, $T_o(k) = 2$ by the rows. Then, Table 8 is obtained. Each column in this table corresponds to different variables and no consistency check is carried out.

Table 8 has an intermediate variable $q_h(k)$. Identify the table having $q_h(k)$ as an output column. Table 3 is identified. Replace each value of $q_h(k)$ in Table 8 by the corresponding rows in Table 3. The resulting table has two rows having the same variable $q_V(k)$, and consistency has to be checked in the following way. First, remove any row having different values of $q_V(k)$, since a variable can not have plural values in a row. Table 9 is obtained. Then, reduce the columns having the com-

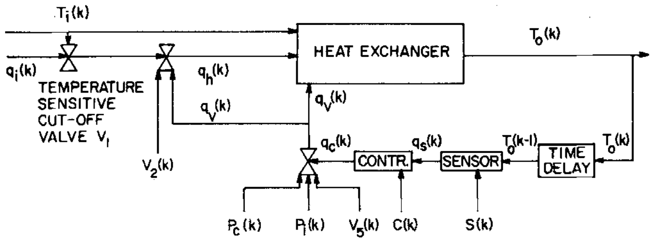

Fig. 2. Improved diagram of cooling system.

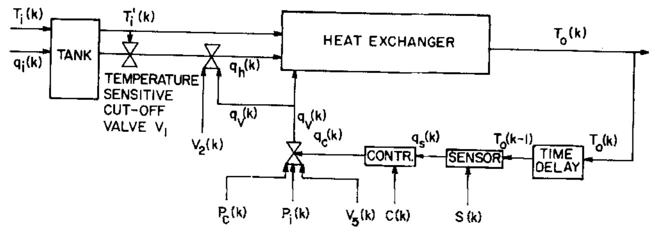

Fig. 3. Final diagram of cooling system.

TABLE 12. CRITICAL TRANSITION TABLE FOR $q_h(k) = -2$ FOR THE SYSTEM IN FIGURE 2

Mode No.	$V_1(k)$	$V_2(k)$	$p_C(k)$	$V_5(k)$	$C(k)$	$S(k)$	$p_i(k)$	$q_i(k)$	$T_i(k)$	$T_o(k-1)$	$q_h(k)$	N
54	—	N	−2	—	—	—	—	—	—	—	−2	1
55	—	N	0	N	—	—	−2	—	—	—	−2	1
56	—	—	—	—	—	—	—	−2	—	—	−2	1
57	N	—	—	—	—	—	—	—	—	—	−2	1

mon variable $q_V(k)$ into a single column by performing *AND* operation of the corresponding entries in the columns. Table 10 is obtained from Table 9.

Repeat the elimination of the intermediate variables in a similar way. We eventually obtain a table having only primary variables and state variables. Such a table is called a critical transition table because it gives all the possible combinations of the primary variables causing the system failure developing from each system state. Each combination is a "Critical Transition Mode" and the set of all the modes is called a critical transition set. We have Table 11 for the cooling system in Figure 1.

SYSTEM IMPROVEMENT BASED ON CRITICAL TRANSITION SETS

Improving System Reliability

Each occurrence of a system failure has to correspond to a mode in the critical transition set. The elimination of dominant modes is important for practical system improvements.

The mode is called an N-event critical transition mode if it includes at least N failure events. Thus, mode 1 in Table 11 is a two-event critical mode since it has two failure events $p_i(k) = -2$ and $V_2(k) = R$. Table 11 lists the number N for each mode.

The one-event modes 27, 28, 33, and 34 can be removed by the installation of a temperature sensitive cutoff valve. The valve cuts off the inflow into the heat exchanger when the temperature $T_i(k)$ is too high. The improved system diagram is shown in Figure 2.

Improving System Availability

We now consider another failure defined by $q_h(k) = -2$ for the system in Figure 2. Using a similar procedure to that used previously, we have the critical transition Table 12 for the failure.

The system unavailability is characterized by the event $T_o(k) = 2$ or $q_h(k) = -2$. Consider reducing the possibility of the one-event modes in Table 12, since the one-event modes in Table 11 have already been removed. This decreases the system unavailability if we assume that we must shut down if the temperature is too high. A feasible policy is as follows: increase the reliability of the cooling water pressure $p_C(k)$ (mode 54), increase the reliability of the actuating pressure $p_i(k)$ (mode 55), and install a tank at the entrance of the system, reducing the possibility of $q_i(k) = -2$ or $T_i(k) = 2$ (modes 56, 57).

The final system is shown in Figure 3. The system is guaranteed to have high availability along with high safety if it is assumed that $T_o(k) = 2$ will be a major cause of an accident.

DISCUSSION AND CONCLUSION

The critical transition set is characterized by the fact that each occurrence of the system failure has to correspond to a mode in the set. It might be possible that a mode in the set is never accompanied by any occurrence of the system failure. In this sense, the critical transition set is regarded as an extended set of the exact failure modes set.

It is this extended nature that makes it possible for us to obtain the set easily and to utilize it for a robust system reliability synthesis.

The proposed method can be generalized so as to handle the system failure such as

$$T_o(k-1) \neq 2 \quad \text{and} \quad T_o(k) = 2 \qquad (1)$$

The critical transition set of the failure can be obtained by eliminating the rows having $T_o(k-1) = 2$ from Table 11.

The refinement of the system failure like Equation (1) would be useful in some cases in removing nonactual dominant modes from the critical transition set. However, too much refinement is not desirable. For example, the first failure

$$T_o(0) \neq 2, \quad T_o(1) \neq 2, \ldots, \quad T_o(k-1) \neq 2, \quad T_o(k) = 2, \quad \text{for some } k \qquad (2)$$

gives critical transition modes exactly the same as those obtained by the conventional fault trees.

An upper bound of the system failure probability can be obtained from the extended nature of the critical transition set when the stochastic property of the primary variables are given, using existing computer programs based on cut set inputs (Fussell, 1972; Vesley, 1970).

ACKNOWLEDGMENT

This work was made possible by Grant ENG 75-17613.

NOTATION

C = internal mode of controller
k = discrete time, integer
p_C = cooling water pressure into cooling water con-

trol valve
p_i = actuating pressure of cooling water control valve
q_C = output of controller
q_h = material flow rate to heat exchanger
q_i = inflow rate of material to be cooled
q_S = output of temperature sensor
q_V = flow rate of cooling water
S = internal mode of temperature sensor
T_i = temperature of material into heat exchanger
T_o = temperature of material out of heat exchanger
V_2 = internal mode of cutoff valve
V_5 = internal mode of cooling water control valve

LITERATURE CITED

Fussell, J. B., and W. E. Vesely, "A New Methodology for Obtaining Cut Sets for Fault Trees," *Trans. Am. Nucl. Soc.*, **15**, 262 (1972).

Fussell, J. B., "Synthetic Tree Model—A Formal Methodology for Fault Tree Construction," *ANCR-1098*, Available from NTIS, Springfield, Va. (1973).

———, "Fault Tree Analysis—Concepts and Techniques," in *Generic Techniques of System Reliability Assessment*, E. Henley and J. Lynn, ed., Nordhoff Publishing Company (1976).

Henley, E. J., and H. Kumamoto, "Comments on Computer-Aided Synthesis of Fault Trees," *IEEE Trans. Reliability*, **R-26** (1977).

Lapp, S. A., and G. J. Powers, "Computer-Aided Synthesis of Fault-Trees," *ibid.*, (1977).

Pollack, S. L. *Decision Tables: Theory and Practice*, Wiley-Interscience, New York (1971).

Salem, S. L., G. E. Apostolakis, and D. Okrent, "A New Methodology for the Computer-aided Construction of Fault Trees," *Ann. Nucl. Energy*, **4**, 433 (1977).

Vesely, W. E., and R. E. Narum, "PREP and KITT; Computer Codes for the Automatic Evaluation of a Fault Tree," IN-1349, Available from NTIS, Springfield, Va. (1970).

Manuscript received January 13, 1978; revision received August 7, and accepted August 25, 1978.

Mass Transfer in Regular Arrays of Hollow Fibers in Countercurrent Dialysis

ISAO NODA
and
CARL C. GRYTE
Department of Chemical Engineering and Applied Chemistry
Columbia University
New York, New York 10027

The concentration profiles in a countercurrent hollow fiber bundle mass exchanger dialyzer is derived, assuming a uniform distribution of fibers in the dialyzer shell. Mass transfer coefficients are obtained as a function of fiber packing density, membrane thickness, membrane material, and solute type.

SCOPE

Dialysis is a membrane separation process which involves diffusive transport of a solute from one fluid to another. Although the solute flux is often accompanied by some volumetric flux across the membrane, driven either by hydraulic or osmotic pressure, we shall focus here only on diffusive transport systems. The input-output response of a continuous dialysis unit using various flow geometries is given by Michaels (1966). The rate of mass transfer of a solute across the membrane during dialysis is expressed in terms of the surface area, the concentration difference across the membrane, and the overall mass transfer coefficient. The magnitude of the overall mass transfer coefficient for a given dialyzer is usually determined experimentally. Dialysis membranes in the form of hollow fibers are gaining considerable attention because, in this configuration, large membrane surface areas are obtained for a given dialyzer volume. The most significant application is in the area of hemodialysis (Klein et al., 1976*a*). Recently, Noda (1976) has proposed highly selective multistaged dialysis separation processes.

Much data are available for the individual polymeric membrane materials (Colton, 1971; Klein et al., 1977). Detailed theoretical descriptions are reported to account for dialysis with planar membrane surfaces (Leonard, 1968), but only simplistic models are available to describe the mass transfer in closely packed hollow fiber bundles. Stevenson (1975) has reported a method for determining the membrane permeability of a single, isolated, hollow fiber. Methods have been proposed by Klein et al. (1977) to characterize bundles of well-separated hollow fibers empirically. A discussion of the problems associated with the characterization of a hollow fiber bundle similar to that used in hemodialysis has been recently given by Klein et al. (1976*a*, 1976*b*). In all studies, however, the procedure has been to reduce the analysis to that of a single isolated fiber [equivalent annulus approximation, Happel (1959)], thereby neglecting the interactions between fibers which become important when the interfiber distances are small.

The aim of this investigation is to establish a theoretical understanding of hollow fiber dialysis systems by using a